Experimental evaluation of Madelung's rules for static magnetization curves

Ermin Rahmanović^a, Simon Steentjes^b, Martin Petrun^a

^a Institute of Electrical Power Engineering, FERI, University of Maribor, Maribor, Slovenia. ^b Institute of Electrical Machines (IEM), RWTH Aachen University, Aachen, Germany.

The behaviour of soft magnetic materials is very nonlinear, making them complex to model mathematically. E. Madelung established three rules governing the behaviour of static magnetization curves within the major loop based on a huge number of measurements. These rules serve as a guide for the implementation of the phenomenological history-dependent hysteresis models. This research work analyzes the adequacy of Madelung's rules by examining purposefully selected and measured magnetization curves of different state-of-the-art non-oriented and grain-oriented electrical steel grades. Early findings confirm the rules' applicability to both non-oriented and grain-oriented magnetic materials.

Keywords: Electromagnetic measurements; Madelung's rules; magnetic hysteresis; static magnetic properties

1. Introduction

To accurately model the nonlinear magnetic behaviour, phenomenological history-dependent hysteresis models (HDHM) are used [1], which fundamentally rely on Madelung's empirical rules [2, 3]. These rules describe how the magnetization history, particularly the reversal points (RP), influences the current magnetization path. Nowadays, Madelung's rules also serve as the fundamental validation criteria for hysteresis models. The presented analysis focuses on the three original rules and evaluates their relevance for modern hysteresis modelling approaches.

2. Results and discussion

The aim of this work is to analyze the adequacy of Madelung's rules based on measured magnetization curves of state-of-the-art non-oriented (NO) and grain-oriented (GO) electrical steel grades. To test all three of the originally proposed Madelung's rules [1], we measured an asymmetric minor loop formed on a first-order reversal curve (FORC) and compared that magnetization curve with a FORC measured from the same RP as in the case with the minor loop. Measurements were performed for a NO sheet with 0.27-mm thickness (NO27) and a GO sheet with 0.27-mm thickness (GO27). The measured magnetization curves are presented in Figure 1.



Figure 1: Measurement of an asymmetric minor loop on a FORC.

The minor loop on the FORC was formed by a second-order reversal curve (SORC) and a third-order reversal curve (TORC). The TORC tends to return exactly to the RP where it originated from. This confirms the validity of the second rule. After completing the minor loop, the magnetization path continues along the FORC. By comparing the FORC from the magnetization path with a minor loop and the FORC measured directly to saturation, it is apparent that there are low deviations between them. This implies that after the completion of the minor loop the magnetization curve continues its path on the curve it was previously on. Consequently, the third rule is also verified. By increasing the magnetic field strength *H* the FORC approaches the saturation region as presented in Figure 2. Finally, this confirms the validity of the first rule.



Figure 2: Asymmetric minor loop on FORC presented up to saturation.

The final presentation will include a comprehensive analysis, also considering the two additional rules proposed by Zirka [3]. The analysis will be supported by specific measurements that will enable an in-depth evaluation of all five rules governing static magnetization curves. Measurements will be performed on three commercial electrical steel grades to demonstrate the applicability of the rules on state-of-the-art materials used in applied engineering. The conclusions will offer insight into scenarios where HDHM can be expected to yield accurate results.

References

- G. Mörée, and M. Leijon, "Review of Hysteresis Models for Magnetic Materials," Energies, vol. 16, no. 9, pp. 3908, 2023-05-05, 2023.
- [2] E. Madelung, "Über Magnetisierung durch schnellverlaufende Ströme und die Wirkungsweise des Rutherford-Marconischen Magnetdetektors," Annalen der Physik, vol. 322, no. 10, pp. 861-890, 1905.
- [3] S. E. Zirka, Y. I. Moroz, P. Marketos, and A. J. Moses, "Congruency-Based Hysteresis Models for Transient Simulation," IEEE Transactions on Magnetics, vol. 40, no. 2, pp. 390-399, 2004-03-01, 2004.