A semi-coupled model of a Fe-Ga cantilever beam with PWL stress-dependent magneto-elastic characteristic

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Energy harvesters allow the deployment of low-power and smart sensors in harsh environments and are an example of local green-oriented power supplies [1, 2]. Here, it is proposed a novel piece-wise linear (PWL) stress-dependent magnetostrictive characteristic, coupled with the Euler-Bernoulli equation, to model a multi-layer Fe-Ga/Al cantilever beam [3], a configuration widely employed in kinetic energy harvesting. The results are compared with experiments.

Keywords: Magnetoelasticity; Energy Harvesting; Structural beams

1. Introduction

Magnetostrictive cantilever beams are effective and reliable for energy harvesting, powering wireless sensors via environmental vibrations. Energy is recovered through kinetic energy harvesters (KEH), exploiting environmental vibrations. A common KEH design is the multi-layer Fe-Ga/Al cantilever beam [3], typically modeled by the Euler-Bernoulli equation while assuming a constant elastic modulus and linear magneto-elastic behavior. Although generalizing these assumptions increases computational complexity, this study introduces a piecewise linear (PWL) stress-dependent B-H model for an accurate yet efficient representation of the conversion process. The approach incorporates the ΔE-effect [4], capturing the influence of the bias magnetic field on the elastic modulus, while maintaining closed-form analytical solutions for simplified analysis and design.

2. Results and discussion

A 1-D Euler-Bernoulli equation (for a homogeneous beam, as sketched in Fig. 1 left) provides the displacement of the neutral axis with respect to its rest position. The Fe-Ga layer is modeled and coupled to E-B equation, using the constitutive relation:

$$B_z = \mu_0 H_z + M_z(H_z, \sigma_{zz}),$$

where M_z depends on the magnetic field and longitudinal stress. A previous work [5] developed a fully coupled nonlinear model to compute B_z. Here, a novel piecewise linear approach is adopted (see Fig.1, right):

$$M_z(H_z) = \begin{cases} \chi_0\left(1-\dfrac{T_0+\sigma}{T^*}\right)H_z & if\ H < H_S(\sigma) \\ M_S & if\ H \geq H_S(\sigma) \end{cases}$$

where $H_s(\sigma)$ is the transition field, χ_0 is the susceptibility at $\sigma = -T_0$ (stress-free state) and M_s is the saturation magnetization. T_0 is the residual compressive stress in Fe-Ga alloys, while T^* is a reference stress identifiable from experiments. This model effectively captures the stress dependence of magnetization within $T^* - T_0 \leq \sigma \leq -T_0$. Fig. 2 compares simulation results performed considering the piece-wise linear model and those with linear and non-linear models. Experimental measurements have been added to prove the approach's goodness, rather than finding the highest energy harvesting performance. The model works fine for both tensile and compressive stress. Finally, it is worth noting that the solution can also be determined analytically.

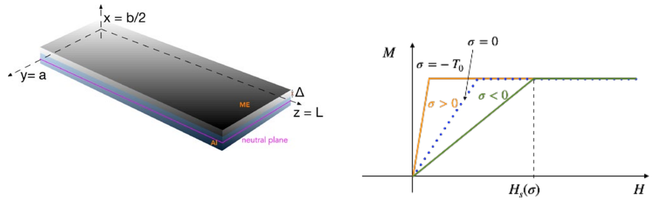

Figure 1: Sketch of a Fe-Ga/Al bi-layer cantilever beam (left). Conceptual sketch of the PWL characteristic (right).

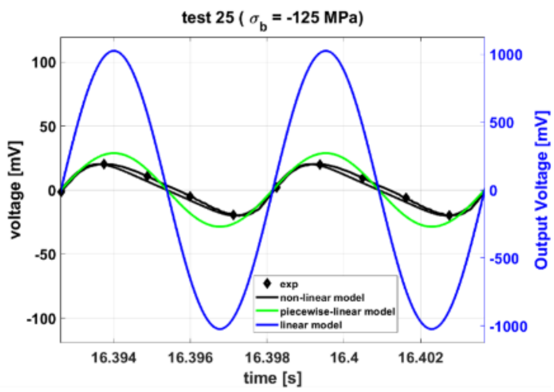

Figure 2: Comparison between experimental (black with diamonds) and simulated (non-linear model in black, linear model in blue, piece-wise linear in green) open circuit output cantilever beam voltage. Please note that different scales have been adopted for experimental, non-linear and piece-wise linear models on one side (left), and linear model predictions on the other (right).

References

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