B-driven Energy-Based model by using a modified Newton-Raphson algorithm

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An inverse (i.e. **B**-driven) Energy-Based hysteresis model is presented. The model is inverted by using a modified Newton-Raphson method. In the vector case, angular points in the path of the **B** field are observed when a cell is activated, and they are found to hinder the convergency of the classical Newton-Raphson method – even equipped with backtracking. The proposed method improves the convergency with respect of the classical algorithm, and it is validated with simulations and experimental measurements performed on a 3.7% Fi-Si steel.

Keywords: Static hysteresis; Energy Based model; Finite Element

1. Introduction

Among existing static hysteresis models, the Energy Based (EB) model has some desirable features, like the fact that it is intrinsically vectorial, the respect of thermodynamic laws and the easiness of identification and implementation. In this model the **H** field is decomposed in a reversible and irreversible component: $\mathbf{H} = \mathbf{H}_r + \mathbf{H}_i$. The reversible part $\mathbf{H}_r$ writes:

$$\mathbf{H}_r = \sum_{k=0} \omega_k \mathbf{q}_k$$

where $\mathbf{q}_k$ is the state of the kth cell and $\{\omega_k\}$ is a partition of the unity. When the **H** field is modified, each state is updated basing on the nonlinear, discontinuously differentiable law:

$$\mathbf{q}_k \leftarrow \begin{cases} \mathbf{H} - \kappa_k \dfrac{\mathbf{H}-\mathbf{q}_k}{||\mathbf{H}-\mathbf{q}_k||} & \text{if } ||\mathbf{H}-\mathbf{q}_k|| > \kappa_k \\ 0 & otherwise \end{cases}$$

The magnetization is computed through an anhysteretic function (usually a Langevin's function):

$$\mathbf{M} = M_{an}(||\mathbf{H}_r||)\frac{\mathbf{H}_r}{||\mathbf{H}_r||}$$

This classical formulation of the EB model is **H**-driven (i.e. given a magnetic field **H**, the flux density **B** is computed). In some cases, the model has to be driven by the **B** field and must therefore be inverted [1], notably when it is used in Finite Element codes with an **A** formulation [2]. This work presents algorithms to invert a vector EB model by using NR method.

2. Results and discussion

Netwon-Raphson's method (NR) is the workhorse for solving nonlinear equations $\mathbf{F}(\mathbf{h}) = 0$. Under some broad hypothesis and close to the solution it has 2nd order convergency. Unfortunately, the convergence of NR's method strongly depends on the starting point, and cannot be guaranteed. For this reason, several improvements have been devised, among which Armijo's rule, also called backtracking [3]. Armijo's rule is a line search method which, at each time step, possibly reduces the step length:

$$\mathbf{h}_{n+1} = \mathbf{h}_n - \alpha_k \mathbf{J}(\mathbf{h}_n)^{-1}\mathbf{F}(\mathbf{h}_n)$$

where $\mathbf{J}(\mathbf{h}_n)$ is the Jacobian matrix. The scalar $\alpha_k \in (0;1]$ is a coefficient which is computed in such a way that the new point $\mathbf{h}_{n+1}$ is closer to the solution than the actual point $\mathbf{h}_n$.

With the purpose of inverting the EB, a NR method with Armijo's rule has been implemented. It is found that Armijo's rule ensures the convergency of NR's method in the scalar case, but it generally fails in the vector case. Our hypothesis to explain this failure is that the EB model ensures continuity but not differentiability. In particular, when a cells is activated ($||\mathbf{H}-\mathbf{q}_k|| = \kappa_k$) we have an angular point in the path of the **B** field – that is, the direction of the path of the **B** field changes abruptly because the cell is "activated" (Fig. 1). This angular point hinders the convergency of NR algorithm – basically because when this happens $\mathbf{J}(\mathbf{h}_n)^{-1}\mathbf{F}(\mathbf{h}_n)$ points to the wrong direction, whatever α_k. Notice that this happens only in the vector case: this explains why Armijo's rule alone works fine in the scalar case, but it does not suffice in the vector case.

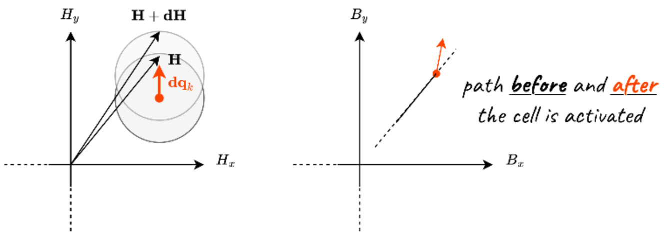

Figure 1: sketch of an angular point in the path of **B** field

In order to improve the convergency of NR method, some modifications have been introduced: i) the maximum step of **h** is bounded in an adaptive way, ii) the Jacobian matrix is recomputed (numerically) at each step of Armijo's iterations, so as to catch an eventual angular point. The inverted EB model has been tested on real measurements performed on a 3.7% Fe-Si steel, with pretty good results in terms of accuracy and of number of iterations required to achieve convergency.

References

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